Math 53, Discussion 107

Monday, March 30, 2020 8:47 AM

1) Find the minimum and Maximum values of the function $f = (x-1)^2 + (y-1)^2$ on the unit disc $x^2 + y^2 \leq 1$
2) Calculate $\int_0^1 \int_{x^{\frac{2}{3}}}^1 x\cos(y^4)\,dy\,dx$
3) ~~Find the area of the region enclosed by the curve $x^2 + xy + y^2 = 1$~~
~~Hint: use the substitution $x = u + v\sqrt{3}, y = u - v\sqrt{3}$~~
4) Let R be the region above the cone $z = \sqrt{3x^2 + 3y^2}$ and below the sphere $z = \sqrt{9 - x^2 - y^2}$. Set up an integral or the volume of this region using the following integration orders:
   A) $dz\,dx\,dy$
   B) $dz\,dr\,d\theta$
   C) $dx\,dy\,dz$
5) A stick of length 1 is randomly dropped on a ruled sheet of paper, where the lines are spaced 2 units apart. What's the probability that the stick touches a line?

Joint probability density functions

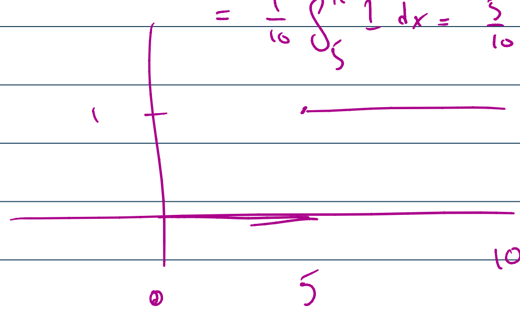
in 1-D I could ask:

1) what is the probability of picking a # between 0 – 10 this is larger than 5.

$$\frac{1}{10}\int_0^{10} (\text{is } x \text{ good?})\,dx = \frac{1}{10}\int_0^{10} f(x)\,dx$$

← probability density function.

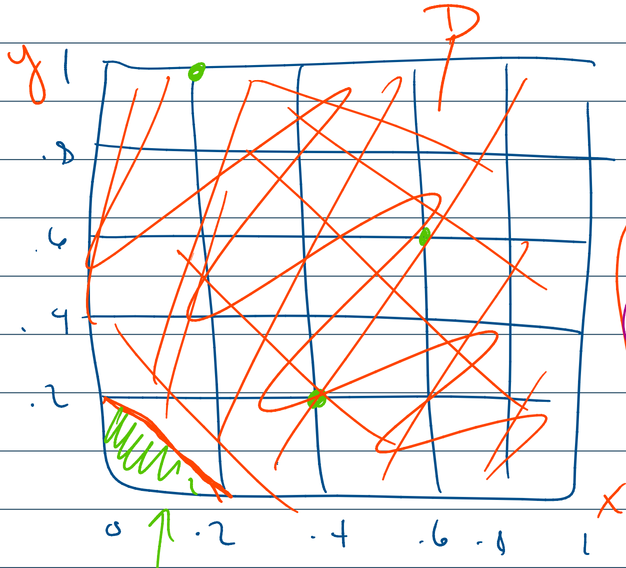
$$= \frac{1}{10}\int_5^{10} 1\,dx = \frac{5}{10} = \frac{1}{2}$$



2) what is the probability of picking x & y between 0 & 1 s.t. $x+y > .2$

D

between 0 & 1 s.t. x+y > .2



$\frac{1}{1}\int_0^1\int_0^1 f(x,y)\,dx\,dy = 98\%$

$f(x,y) = \begin{cases} 1 & \text{if } x+y > .2 \\ 0 & \text{if } x+y \le .2 \end{cases}$

$x+y > .2 \Rightarrow y > .2 - x$

$= \iint_D 1\,dx\,dy$

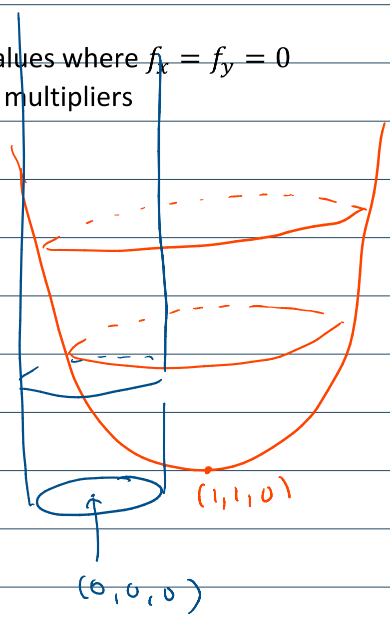
$\frac{1}{2}(.2)(.2)$

$\frac{1}{2}\cdot\frac{2}{10}\cdot\frac{2}{10} = \frac{2}{100} = .02$

Find the minimum and Maximum values of the function $f = (x-1)^2 + (y-1)^2$ on the unit disc $x^2 + y^2 \le 1$

Extreme value theorem: a continuous function on a closed set attains its max and min values

First check in interior by looking for values where $f_x = f_y = 0$
Second: check boundary via Lagrange multipliers



- no local min or max
- use Lagrange multipliers

$\begin{cases} 2(x-1) = \lambda 2x \\ 2(y-1) = \lambda 2y \\ x^2+y^2 = 1 \end{cases}$

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ min

$\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ max

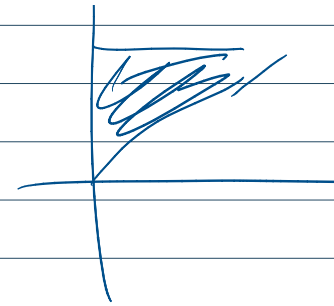
Calculate $\int_0^1 \int_{x^{\frac{2}{3}}}^1 x\cos(y^4)\,dy\,dx$ (D)

D =

Calculate $\int_0 \int_{x^{\frac{2}{3}}} x\cos(y^4)\,dy\,dx$

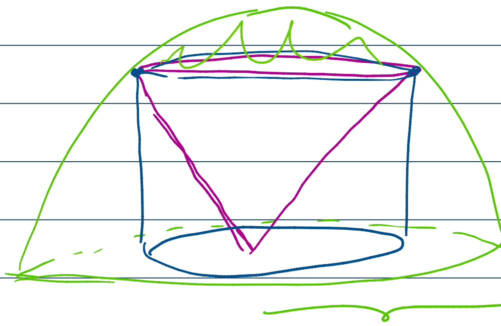
Swap order of integration

D =



1) Let R be the region above the cone $z = \sqrt{3x^2 + 3y^2}$ and below the sphere $z = \sqrt{9 - x^2 - y^2}$. Set up an integral or the volume of this region using the following integration orders:

A) $dz\,dx\,dy$
B) $dz\,dr\,d\theta$
C) $dx\,dy\,dz$ ←



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$\sqrt{3x^2 + 3y^2} = \sqrt{9 - x^2 - y^2}$

$3x^2 + 3y^2 = 9 - x^2 - y^2$

$4x^2 + 4y^2 = 9$

$x^2 + y^2 = \left(\frac{3}{2}\right)^2$

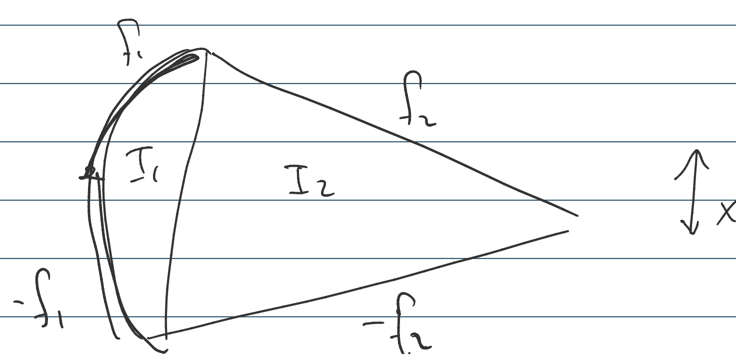
$y = \pm\sqrt{\frac{9}{4} - x^2}$

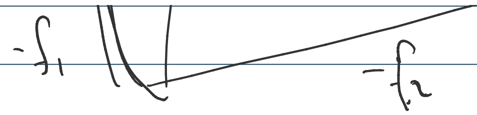
a) $R = \{(x,y,z) \text{ s.t. } \sqrt{} \le z \le \sqrt{},$

$$\int_{-3/2}^{3/2} \int_{-\sqrt{\frac{9}{4} - y^2}}^{\sqrt{\frac{9}{4} - y^2}} \int_{\sqrt{3x^2+3y^2}}^{\sqrt{9-x^2-y^2}} 1 \; dz\,dx\,dy$$

b) $dz\,dr\,d\theta$

$$\int_0^{2\pi} \int_0^{3/2} \int_{\sqrt{3}\,r^2}^{\sqrt{9-r^2}} 1 \cdot r \; dz\,dr\,d\theta$$





-f1
-f2